Control Logic to Track Outputs of a Command Generator

Thomas L. Trankle*
Systems Control, Inc., Palo Alto, Calif.
and
Arthur E. Bryson Jr.†
Stanford University, Stanford, Calif.

A procedure is presented for synthesizing time-invariant control logic to cause the outputs of a linear plant to track the outputs of an unforced (or randomly forced) linear dynamic system. The control logic uses feedforward of the reference system state variables and feedback of the plant state variables. The feedforward gains are obtained from the solution of a linear algebraic matrix equation of the Lyapunov type. The feedback gains are the usual regulator gains, determined to stabilize (or augment the stability of) the plant, possibly including integral control. The method is applied here to the design of control logic for: 1) a second-order servomechanism to follow a linearly increasing (ramp) signal, 2) an unstable third-order system with two controls to track two separate ramp signals, and 3) a sixth-order system with two controls to track a constant signal and an exponentially decreasing signal (aircraft landing-flare or glide-slope-capture with constant velocity).

Introduction

As air traffic increases, there is increasing pressure to fly precisely specified paths in space to avoid collisions. Often these specified paths can be made up of simple straight arcs, circular arcs, and "exponential fillets." To fly such paths requires accurate navigation and a scheme to control the airplane to stay near the specified path. This paper addresses the latter problem, which is a type of "model following" problem, discussed in the control literature.[1-6]

Linear finite-state, time-invariant models with no forcing functions and nonzero initial conditions can often be used to generate the specified paths. Such a model may be regarded as a "command generator." If such a model is modified by adding random forcing functions, then the states of this "target" model may be estimated using measurements of the target output, and improved tracking of the target is then possible. A typical application is causing a radar antenna to track a moving airplane.[7]

It is shown in this paper that excellent model-following, i.e., tracking of the signals from a command generator (or of a target), can be obtained by using feedforward of the model states (or estimated model states). This method yields significantly better model-following than previous methods that depend on feedback of integrals of the model-following error.[2,5]

Statement of the Problem

Tracking an Unforced Model

Given: 1) the *plant* matrices A, B, C where

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

2) the *model* matrices A_m, C_m where

$$\dot{x}_m = A_m x_m \tag{3}$$

$$y_m = C_m x_m \tag{4}$$

3) the plant state $x(t)$, or the plant output $y(t)$, and 4) the model initial conditions, $x_m(0)$; find the plant input $u(t)$ so that the plant output $y(t)$ follows the model output $y_m(t)$ with an error that tends to zero as time increases. Assume y, u, y_m all have the same dimension.

Tracking a Randomly Forced Model

Given: 1) same as stated previously, 2) a stochastic "target" model,

$$\dot{x}_m = A_m x_m + w_m \tag{5}$$

where w_m is a white noise, 3) same as 3 given earlier, and 4) observations of the target,

$$y_m = C_m x_m + v_m \tag{6}$$

where v_m is white noise; find the plant input $u(t)$ so that the plant output $y(t)$ follows the target output $y_m(t)$ on the average. Assume y, u, y_m all have the same dimension.

Design of Control Logic

Feedforward Gains

Let

$$u = Ux_m + \delta u \tag{7}$$

$$x = Xx_m + \delta x \tag{8}$$

where U and X are constant matrices to be determined so that

$$y = y_m + \delta y \tag{9}$$

and

$$\delta y \to 0 \tag{10}$$

sufficiently rapidly. Substituting Eqs. (2, 4, and 8) into Eq. (9) yields

$$(CX - C_m)x_m + C\delta x = \delta y \tag{11}$$

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*Engineer. Member AIAA.
†Paul Pigott Professor of Engineering, Dept. of Aeronautics/Astronautics. Fellow AIAA.

Substituting Eqs. (3, 5, and 8) into Eq. (1) yields

$$XA_m + \delta\dot{x} = AXx_m + A\delta x + BUx_m + B\delta u \tag{12}$$

If we can choose X and U so that

$$CX = C_m \tag{13}$$

$$AX + BU = XA_m \tag{14}$$

then it follows from Eqs. (11) and (12) that

$$\delta\dot{x} = A\delta x + B\delta u \tag{15}$$

$$\delta y = C\delta x \tag{16}$$

where Eq. (8) implies

$$\delta x(0) = x(0) - Xx_m(0) \tag{17}$$

If the plant is stable and $\delta u = 0$, it follows that $\delta x \to 0$, hence $\delta y \to 0$ and $y \to y_m$ from Eq. (9). A block diagram of the control logic is shown in Fig. 1.

Feedback Gains

If the plant is unstable, or if δy does not die out sufficiently rapidly, stabilization logic, or stability augmentation logic, may be designed using quadratic synthesis. The feedback gain matrix K in

$$\delta u = -K\delta x \tag{18}$$

may be determined by minimizing

$$J = \frac{1}{2}\int_0^\infty (\delta y^T M \delta y + \delta u^T N \delta u)\,dt \tag{19}$$

subject to Eqs. (15) and (16), where M and N are weighting matrices; M is positive semidefinite while N is positive definite. The gain matrix K may also be determined by pole placement techniques.[8]

In terms of the plant and model states, the control logic becomes, from Eqs. (7, 8, and 18)

$$u = Ux_m - K(x - Xx_m) \tag{20a}$$

or

$$u = (U + KX)x_m - Kx \tag{20b}$$

Note the feedforward gain on x_m is modified by the matrix KX when stability augmentation ($K \neq 0$) is used. A block diagram of the control logic is shown in Fig. 2.

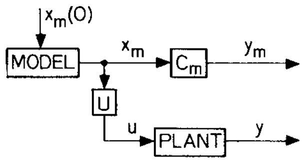

Fig. 1 Model-following with a stable plant.

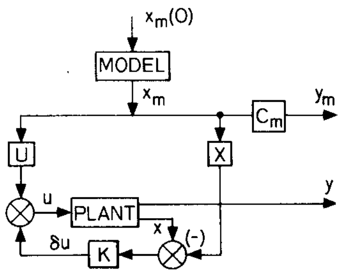

Fig. 2 Model-following with stability augmentation, plant state observed.

Integral-Error Feedback

In general, integral-error feedback[9,10] should be added to the logic to compensate for uncertainties in the plant and model matrices and unmodeled disturbances. To do this, introduce additional states ξ where

$$\dot{\xi} = y - y_m \equiv \delta y \equiv C\delta x \tag{21}$$

and modify Eq. (19) to implement integral quadratic penalties on ξ (rather than δy):

$$J = \frac{1}{2}\int_0^\infty (\xi^T M \xi + \delta u^T N \delta u)\,dt \tag{22}$$

Note that the addition of integral-error feedback does not affect the feedfoward gains.

Incomplete State Information for the Plant

If only $z(t)$, a set of plant measurements, is available, an estimate of the plant state $\hat{x}$ may be generated on-line, for example

$$\dot{\hat{x}} = A\hat{x} + Bu + L(z - Hx),\quad \hat{x}(0) = 0 \tag{23}$$

where the gain matrix L is determined by observer or Kalman-filter theory. $\hat{x}$ is then used in place of x in Eq. (20). A flow chart of the control logic for this case with stability augmentation is shown in Fig. 3.

Tracking a Randomly Forced Model with Only Model Outputs Available

When tracking an external model (or "target") whose inputs and initial conditions are not known, we regard the inputs and initial conditions as random and generate estimates of the model states using only observations of the target outputs. If a target model is known or approximated, then we

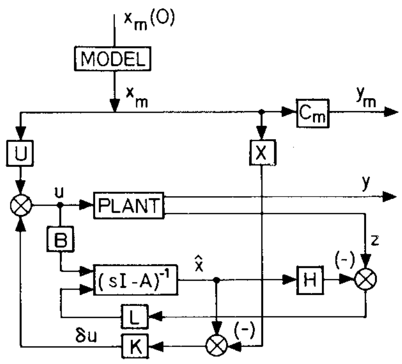

Fig. 3 Model-following with stability augmentation, plant state estimated.

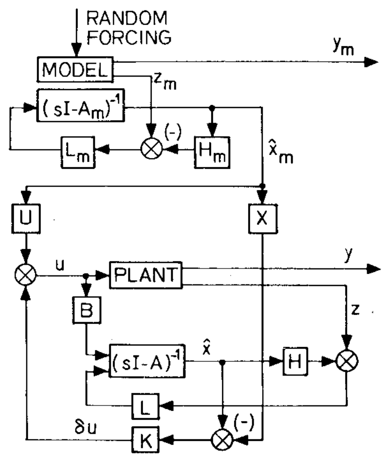

Fig. 4 Following a randomly forced external model with stability augmentation, both model and plant states estimated.

might use

$$\dot{\hat{x}}_m = A_m \hat{x}_m + L_m (y_m - C_m \hat{x}_m), \quad \hat{x}_m(0) = 0 \tag{24}$$

where the gain matrix L_m is determined using observer or Kalman-filter theory. $\hat{x}_m$ is then used in place of x_m in Eq. (20). A block diagram for tracking with incomplete state information on the plant is shown in Fig. 4.

In transfer function nomenclature, Eq. (20) becomes

$$u(s) = (U+KX)(sI - A_m + L_m C_m)^{-1} L_m y_s(s) - Kx(s) \tag{25}$$

which, in turn, implies

$$y(s) = C(sI - A + BK)^{-1} B(U+KX)$$
$$\times (sI - A_m + L_m C_m)^{-1} L_m y_m(s) \tag{26}$$

The use of a plant state estimator does not affect the transfer functions in Eq. (26) but does introduce additional dynamics into the transient tracking errors for nonzero initial conditions.

It is straightforward to show that, in this case

$$y = \hat{y}_m + \delta y \tag{27}$$

where

$$\hat{y}_m \triangleq C_m \hat{x}_m \tag{28}$$

$$\delta y = C\delta x \tag{29}$$

$$\delta \dot{x} = (A - BK)\delta x - XL_m (y_m - \hat{y}_m) \tag{30}$$

Equations (27-30) show that, as long as $y_m - \hat{y}_m$ is small, $y - y_m$ will be small.

Characteristics of the Control Logic

Relationship to Nonzero Set Point Control

Nonzero set points[9,10] represent a special case where the model is simply

$$\dot{x}_m = 0, \; x_m(0) = y_d = \text{desired outputs} \tag{31}$$

$$y_m = x_m \tag{32}$$

In the present nomenclature, $A_m = 0$, $C_m = I$, so that Eqs. (11) and (12) simplify to

$$CX = I \tag{33}$$

$$AX + BU = 0 \tag{34}$$

Relationship to "Perfect" Model-Following

For so-called "perfect" model-following, $\delta y = 0$ in Eq. (9), i.e., no transient error is permitted. This constraint often requires impossibly large control amplitudes and the synthesis methods presented[1] often destabilize the plant (those plant modes not involved in the model-following). In the nomenclature of this paper, perfect model-following is obtained by use of

$$u = (CB)^{-1}(C_m A_m x_m - CAx) \tag{35}$$

Comparing Eq. (35) with Eq. (20) shows that the feedforward gain on x_m and the feedback gain on x are distinctly different.

Relationship to Tracking by Use of Multiple Integral-Error Feedback

Another method for tracking has been proposed.[2] However, only models with outputs that are polynomials in time were treated. Essentially it uses feedforward of model states (using different gains than proposed here) and multiple integral-error feedback. However, the feedforward gains of Ref. 2 do not give the correct asymptotic control signals so they must be adjusted by building up nonzero integral-error states. This takes time and, in general, produces larger transient errors with longer decay times than are obtained with the control logic presented here. For internal models, no integral-error feedback is required, with the control logic proposed here, to obtain exact asymptotic tracking. For randomly forced external models, only single integral-error feedback is needed, and then only to account for the inevitable deviations of the real plant and real target from the mathematical models of the plant and target used to design the control logic.

Design Examples

Example A: Second-Order Servo to Track a Ramp Signal

Consider the classical servomechanism problem where we wish to track a ramp (linearly increasing) signal. The plant and model equations are

Plant

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_2 + u, \quad y = x_1 \tag{36a}$$

Model

$$\dot{x}_{m_1} x_{m_2}, \quad \dot{x}_{m_2} = 0, \quad y_m = x_{m_1} \tag{36b}$$

Solving Eqs. (13) and (14) for this case yields,

$$U = [0,1] \qquad X = \begin{bmatrix} 1, 0 \\ 0, 1 \end{bmatrix} \tag{37}$$

If we augment the stability of the plant by placing the closed-loop poles at $s = 2 \pm 2j$, then $K = [8, 3]$. This gives the control logic

$$u = x_{m_2} - 8(y - y_m) - 3(x_2 - x_{m_2}) \tag{38}$$

If we were tracking an external nearly constant velocity (x_{m_2}) target with only target displacement y_m and plant displacement y available, estimates of x_2 and x_{m_2} could, for example, be obtained by reduced order observers,[8]

$$\dot{\hat{x}}_2 = -\hat{x}_2 + u + l(\dot{y} - \hat{x}_2) \tag{39}$$

$$\dot{\hat{x}}_{m_2} = l_m(\dot{y}_m - \hat{x}_{m_2}) \tag{40}$$

If we choose to place both of these observer poles at $s = -2$, then $l = 1$, $l_m = 2$. The transfer function from y_m to y is then

$$\frac{y(s)}{y_m(s)} = \frac{16(s+1)}{(s+2)(s^2+4s+8)} \tag{41}$$

$$\frac{y(s) - y_m(s)}{y_m(s)} = -\frac{(s+6)s^2}{(s+2)(s^2+4s+8)} \tag{42}$$

Example B: Unstable Third-Order System to Track Two Separate Ramp Signals

Consider a third-order unstable plant (eigenvalues at $s = -1,2,3$) with two controls, where we wish to track two separate ramp signals:

Plant

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{43}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{44}$$

Model

$$\begin{bmatrix} \dot{x}_{m_1} \\ \dot{x}_{m_2} \\ \dot{x}_{m_3} \\ \dot{x}_{m_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{m_1} \\ x_{m_2} \\ x_{m_3} \\ x_{m_4} \end{bmatrix} \tag{45}$$

$$\begin{bmatrix} y_{m_1} \\ y_{m_2} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{m_1} \\ x_{m_2} \\ x_{m_3} \\ x_{m_4} \end{bmatrix} \tag{46}$$

This example was used[2] where all model and plant states were assumed available. It was solved using different feedforward gains, double integral-error feedback, and pole-placement for stabilization.

Solving Eqs. (13) and (14) for this case and stabilizing by minimizing

$$J = \frac{1}{2} \int_0^\infty [(\delta u_1)^2 + (\delta u_2)^2] \mathrm{d}t \tag{47}$$

yields

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0, 1, & -1, & -1 \\ 6, 11, & -10, & -10 \end{bmatrix} \begin{bmatrix} x_{m_1} \\ x_{m_2} \\ x_{m_3} \\ x_{m_4} \end{bmatrix} - \begin{bmatrix} 5.29, & 3.24, & -2.06 \\ -8.82, & -2.06, & 6.76 \end{bmatrix} \begin{bmatrix} x_1 - x_{m_1} - x_{m_2} + x_{m_3} + x_{m_4} \\ x_2 - x_{m_2} + x_{m_4} \\ x_3 + x_{m_2} - x_{m_3} - x_{m_4} \end{bmatrix} \tag{48}$$

where the stabilized plant has poles at $s = -1, -2, -3$.

The control logic, Eq. (48), has an advantage over the logic in Ref. 2 in that no integral-error feedback is needed for an internal model to obtain exact asymptotic tracking. Furthermore, the transient tracking error is smaller and dies out faster if the target velocities change abruptly (as in Figs. 7 and 8 of Ref. 2).

If we were tracking two external nearly constant velocity targets with only the target displacements (y_{m_1} and y_{m_2}) available, then estimates of the target velocities, $\hat{x}_{m_2}$ and $\hat{x}_{m_4}$, could be obtained from two first-order observers.

$$\dot{\hat{x}}_{m_2} = l_{m_1} (\dot{y}_{m_1} - \hat{x}_{m_2}) \tag{49}$$

$$\dot{\hat{x}}_{m_4} = l_{m_2} (\dot{y}_{m_2} - \hat{x}_{m_4}) \tag{50}$$

If we choose to place both observer poles at $s = -2$, then $l_{m_1} = l_{m_2} = 2$. The control logic is the same as Eq. (48) except that (x_{m_1}, x_{m_3}) are replaced by (y_{m_1}, y_{m_2}) and (x_{m_2}, x_{m_4}) are replaced by $(\hat{x}_{m_2}, \hat{x}_{m_4})$.

If additional tracking accuracy is required, integral error feedback could be used. In this case two states, x_4 and x_5, are added, where

$$\dot{x}_4 = y_1 - y_{m_1} \triangleq \delta y_1 = \delta x_1 + \delta x_3 \tag{51}$$

$$\dot{x}_5 = y_2 - y_{m_2} \triangleq \delta y_2 = \delta x_2 + \delta x_3 \tag{52}$$

In place of Eq. (47) we use

$$J = \frac{1}{2} \int_0^\infty \{A[(x_4)^2 + (x_5)^2] + (\delta u_1)^2 + (\delta u_2)^2\} \mathrm{d}t \tag{53}$$

and search on A (a scalar) until the five closed-loop eigenvalues lie roughly in the region $-3 < Rls < -1$. This was achieved with $A = 10$ and gave eigenvalues $s = (-1.62 \pm 1.64j, -2.63, -2.66 \pm 2.65j)$. The corresponding control logic is given by

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0, & 1, & -1, & -1 \\ 6, 11, & -10, & -10 \end{bmatrix} \begin{bmatrix} y_{m_1} \\ \hat{x}_{m_2} \\ y_{m_2} \\ \hat{x}_{m_4} \end{bmatrix} - K \begin{bmatrix} x_1 - y_{m_1} - \hat{x}_{m_2} + y_{m_2} + \hat{x}_{m_4} \\ x_2 - \hat{x}_{m_2} + \hat{x}_{m_4} \\ x_3 + \hat{x}_{m_2} - y_{m_2} - \hat{x}_{m_4} \\ x_4 \\ x_5 \end{bmatrix} \tag{54}$$

where

$$K = \begin{bmatrix} 9.80, 4.20, -2.03, 2.80, -1.46 \\ -2.83, -0.031, 4.63, 1.46, 2.80 \end{bmatrix} \tag{55}$$

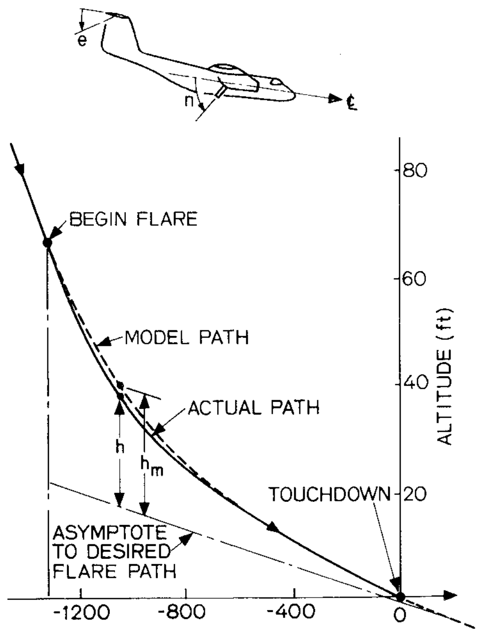

Fig. 5 Altitude vs time for airplane flare maneuver.

$\hat{x}_{m_2}$, $\hat{x}_{m_4}$ are obtained from Eqs. (49) and (50), and x_4, x_5 from

$$\dot{x}_4 = y_1 - y_{m_1} \tag{56}$$

$$\dot{x}_5 = y_2 - y_{m_2} \tag{57}$$

The control logic, Eqs. (54-57), uses only single integral-error feedback and does not assume that the target velocities are available, in contrast to the logic in Ref. 2 where double integral-error feedback is used and the target velocities, as well as displacements, are assumed available.

Example C: Aircraft Landing Flare or Glide-Slope-Capture with Constant Velocity

Consider the rigid-body model of the longitudinal motions of a powered lift STOL airplane[11] flying at sea level at 110 ft/s^{-1}; flight path angle = −1.0 deg (see Fig. 5):

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} v \\ \gamma \\ \theta \\ q \\ h \end{bmatrix} = \begin{bmatrix} -0.0397 & -0.280 & -0.282 & 0 & 0 \\ 0.135 & -0.538 & 0.538 & 0.0434 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0207 & 0.441 & -0.441 & -1.41 & 0 \\ -0.017 & 1.92 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v \\ \gamma \\ \theta \\ q \\ h \end{bmatrix} + \begin{bmatrix} -0.0052 & -0.102 \\ 0.031 & 0.037 \\ 0 & 0 \\ -1.46 & -0.066 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ n \end{bmatrix} \tag{58a}$$

$$y = \begin{bmatrix} v \\ h \end{bmatrix} \tag{58b}$$

where

v = change in velocity, ft/s
γ = change in flight path angle, deg
θ = change in pitch angle, deg
q = pitch angular velocity, deg/s
h = change in altitude, ft
e = change in elevator angle, deg
n = change in engine nozzle angle, deg

We wish to track a commanded exponentially decreasing δh signal with $u=0$, i.e., the model is

$$\dot{v}_m = 0; \quad v_m(0) = 0 \tag{59a}$$

$$\dot{h}_m = -(1/\tau)h_m; \quad h_m(0) \text{ specified} \tag{59b}$$

$$y_m = \begin{bmatrix} v_m \\ h_m \end{bmatrix} \tag{59c}$$

The interpretation of this model as a smooth landing-flare or glide-slope-capture maneuver is shown in Fig. 5.

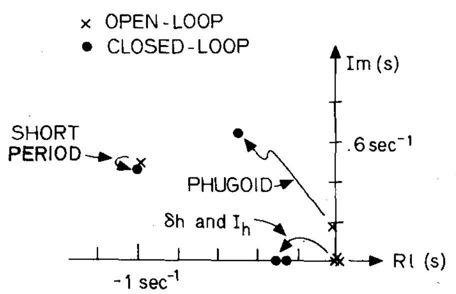

Fig. 6 Open-loop and closed-loop eigenvalues of airplane.

Solving Eqs. (13) and (14) for this case with $\tau = 3.6$ s, and stabilizing by minimizing

$$J = \frac{1}{2}\int_0^\infty \left[\left(\frac{\delta v}{2}\right)^2 + \left(\frac{\delta h}{2}\right)^2 + \left(\frac{I_h}{6}\right)^2 + (\delta e)^2 + \left(\frac{\delta h}{6}\right)^2\right]\mathrm{d}t \tag{60}$$

where $\dot{I}_h = \delta h$, so I_h is the integral of altitude error, we have

$$\begin{bmatrix} e \\ n \end{bmatrix} = \begin{bmatrix} 0.458 \\ -5.03 \end{bmatrix}\gamma_m - K\begin{bmatrix} v \\ \gamma - \gamma_m \\ \theta - 0.822\gamma_m \\ q + 0.228\gamma_m \\ h - h_m \\ \int_0^t (h - h_m)\,\mathrm{d}t \end{bmatrix} \tag{61}$$

where $\gamma_m'(\dot{h}_m/V) \equiv -(h_m/\tau V)$, $V = 110$ ft s^{-1}, and

$$-K = \begin{bmatrix} 0.558 & 2.58 & 1.68 & 0.891 & 1.032 & 0.166 \\ 3.50 & 1.15 & 0.503 & 0.510 & 0.289 & -.0897 \end{bmatrix} \tag{62}$$

Figure 6 shows the open-loop and closed-loop eigenvalues of the system.

A simulation of the airplane [Eqs. (58)] using approach trim conditions as initial conditions gives the results shown in Figs. 5, 7, and 8. Table 1 lists approach trim states x_a, reference desired touchdown states x_r, initial commanded states $x(0)$, and initial perturbations $\delta x(0)$. The initial conditions are related by

$$x_a = x_r + x(0) + \delta x(0) \tag{63}$$

The reference trim state has an elevator angle of −9.6 deg and a nozzle angle of −55.4 deg. The tracking of the model flare path is excellent, and the excursions of controls and states are well within acceptable bounds.

Summary

Using state variable descriptions of a linear time-invariant plant and a linear time-invariant unforced model, a feed-forward plus feedback logic is developed which causes the plant outputs to track the model outputs with initial error transients that depend only on the plant dynamics. Model states (or estimated model states) are fed forward to the plant. If stability augmentation is required, then observed or estimated plant states must be fed back and feedforward gains

Table 1 Flare maneuver initial conditions

	x_a	x_r	$x(0)$	$\delta x(0)$
v	110	110	0	0
γ	−7.5	−1.0	−6.5	0
θ	−4.5	1.3	−5.34	−0.46
q	0	0	1.48	−1.48
h	67.6	22.7	44.9	0

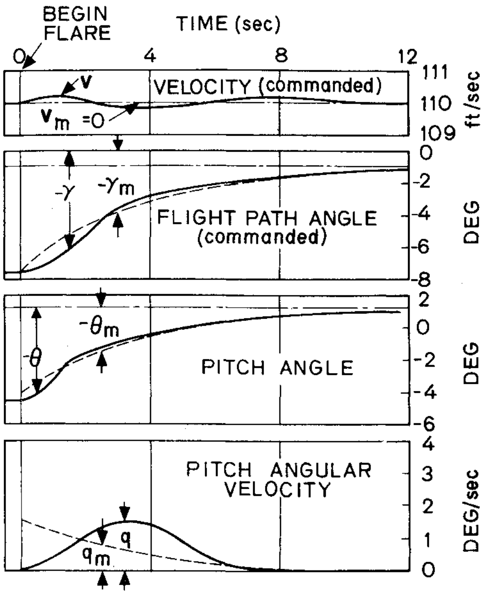

Fig. 7 Velocity, flight path angle, pitch angle, and pitch angular velocity vs time for STOL airplane flare maneuver.

must be modified. Quadratic synthesis may be used to design the feedback gains, and observers or filters may be used to estimate unmeasured plant and target states. Use of feedforward of model states yields faster and more accurate tracking than methods which depend on feedback of integrals of the tracking error.

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References

[1]Erzberger, H., "On Use of Algebraic Methods in Analysis and Design of Model-Following Control Systems," NASA TN D-4663, July 1968.

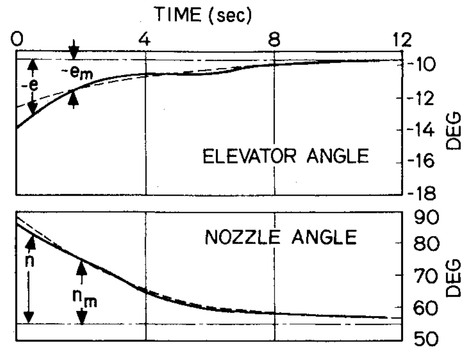

Fig. 8 Elevator angle and nozzle angle vs time for STOL airplane flare maneuver.

[2]Bradshaw, A. and Porter, B., "Design of Linear Multivariable Continuous-Time Tracking Systems Incorporating Feedforward and Feedback Controllers," *International Journal of Systems Science,* Vol. 6, Nov. 1975, pp. 1021-1032.

[3]Tyler, J.S., Jr., "The Characteristics of Modeling Following Systems as Synthesized by Optimal Control," *IEEE Transactions on Automatic Control,* Vol. AC-9, Oct. 1964, pp. 485-498.

[4]Preusche, G., "A Two-Level Model Following Control System and Its Application to the Power Control of a Steam-Cooled Fast Reactor," *Automatica,* Vol. 8, March 1972, pp. 143-151.

[5]Tiroshi, I. and Elliot, J.R., "Explicit Model Following Control Scheme Incorporating Integral Feedback," *Journal of Aircraft,* Vol. 11, June 1974, pp. 364-366.

[6]Kreindler, E. and Rothschild, "Model-Following in Linear-Quadratic Optimization," *AIAA Journal,* Vol. 14, July 1976, pp. 835-842.

[7]Fitts, J.M., "Aided Tracking as Applied to High Accuracy Pointing Systems," *IEEE Transactions on Aerospace and Electronics Systems,* Vol. 9, May 1973, pp. 350-368.

[8]Kwakernaak, H. and Sivan, R., *Linear Optimal Control Systems,* Wiley-Interscience, New York, 1972, pp. 270-280.

[9]Holley, W.E. and Bryson, A.E., Jr., "Multi-Input, Multi-Output Regulator Design for Constant Disturbances and Non-Zero Set Points with Application to Automatic Landing in a Crosswind," Stanford University Department Aero/Astro, Rept. 465, Aug. 1973.

[10]Bryson, A.E., Jr. and Luenberger, D.G., "The Synthesis of Regulator Logic Using State Variable Concepts," *Proceedings IEEE,* Vol. 58, Nov. 1970, pp. 1803-1811.

[11]Cleveland, W.B., Vomashe, R.F., and Sinclair, S.R.M., "Augmentor Wing Jet STOL Research Aircraft Digital Simulation Model," NASA TM X-62 149, April 1972.